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REVIEWS

Dynamique des Fluides. By I. L. RYHMING. Presses Polytechniques Romandes, Lausaune, 1985. 448 pp. SF 130.00.

Fluid mechanics owes as much to the French as to any nation. One thinks of the theoretical and mathematical work of Laplace, Cauchy, Savart, Boussinesq, Poincaré and the experimental contributions by Pitot, Chèzy, Darcy, Hugoniot and Bénard. Currently there are probably more fluid-mechanics research groups in France than in any other western European country, as readers of this *Journal* may be becoming aware. Yet there are only a few textbooks of fluid mechanics written by the French or in the French language (e.g. Brun & Martinot-Lagarde, *Mécanique des Fluides*). There are a few important books on the mathematical foundations of fluid mechanics, e.g. by Cabannes and Germain, but none that combine the theoretical and experimental aspects of the subject, in the style say of Schlichting's *Boundary Layer Theory*. Surprisingly few of the texts written in English or German have been translated into French. So it is an important event when a comprehensive modern textbook on fluid mechanics is written in French. However, that is only a minor attribute of this interesting text, written by Inge Ryhming, a Swede working at the Dynamique des Fluides, Ecole Polytechnique Fédérale de Lausanne. This text forms the basis of a two-year course starting in the second year of a specialized engineering course, in the European system of engineering education. It will be of more interest to aeronautical engineers. The mathematical level is appropriate for students who have taken two years of mathematical methods.

The first chapter briefly introduces the physical ideas of the continuum approach, dimensions, and compressibility. The author touches on the important distinctions between streamlines and particle paths, and on the physical insights gained by switching attention from one to the other. There is one brief paragraph on the singular points in the flow field, i.e. where, in some frame, the velocity components are zero. This point (following Poincaré) ought to be essential in any text or course on fluid mechanics because identifying and localizing these singular points provides an instructive and economical way to define and understand the really complex flows that are being computed and measured in modern fluid-mechanical problems. (See Tobak & Peake, *Ann. Rev. Fluid Mech.* vol. 14, 1982, p. 61).

The second chapter is an extensive account of the kinematical analysis and boundary conditions for fluid flow. The nice diagrams of three-dimensional flows are reminiscent of the best US textbooks.

There is a very brief third chapter on the balance of forces over a material element and over a control surface.

Chapter 4 gives the equations of inviscid fluid flows, and Kelvin and Helmholtz's theorems, with two illustrative examples of inviscid rotational flows, in a converging pipe and around a cylinder whose axis is perpendicular to the vorticity of the oncoming flow. [There is one reference, and perhaps there should be more, to the well-known fluid-mechanics films, e.g. by Shapiro, on Vorticity, and to Van Dyke's *Album of Fluid Motion.*]

Chapters 5, 6 and 7 from pages 101 to 378, are the meat of the book, on inviscid incompressible flow, viscous incompressible flow, and inviscid compressible flow. Chapter 5 gives a sound basis for the study of aerofoils, bubbles, vortex sheets and wave motion. Unlike many engineering texts, this one contains formal statements of the mathematical problems to be solved and, where appropriate, the uniqueness of solution is pointed out. There are only indications as to how computations might be performed. The author might have pointed out that in the computational approach to fluid-dynamical problems, it is essential for the mathematical problem to be defined without ambiguity and that the question of uniqueness is by no means academic.

Chapter 6 introduces the viscous equations of motion with references to their originators, both the familiar, Navier and Stokes, and the less familiar St. Venant and Poisson. Dimensionless groups and the concept of similitude are derived from the governing equations. After giving a few of the simplest exact and low-Reynolds-number solutions of the equations (pipes, oscillating plane, lubrication bearing), laminar boundary layers are treated at length using order-of-magnitude estimates, similarity theory, and approximate methods. It is nice to see in an engineering textbook the use of van Dyke's model equation ($\epsilon f''+f' = a; f(0) = 0, f(1) = 1$, as $\epsilon \rightarrow 0$) to explain the mathematical basis of boundary-layer theory.

This Chapter follows the usual approach of engineering texts in not *discussing* the effect of a boundary as a source of vorticity which diffuses outwards or inwards (although there is a formal derivation of this result). Surely this is the best way of explaining the Stokes oscillating plate, the process of separation and the formation of wakes? Also the similarity and differences between momentum and scalar transfer become much easier to understand. It is also a revealing way of explaining why the turbulent boundary layer separates much later than a laminar one (because in an adverse pressure gradient the reverse vorticity is diffused more rapidly than in a laminar flow).

Chapter 6 also includes a good brief qualitative introduction into the transition of laminar to turbulent flows and fully developed turbulent flows, in particular the randomness of turbulent flows and their multiscale nature. The main direction is towards deriving equations for the mean flow and Reynolds shear stress in turbulent boundary layers, using the mixing-length concept of turbulence.

Again, I would have expected some cautionary remarks before and after such a treatment, emphasizing that turbulent transfer is generally *not* a local phenomenon either in physical space, as is assumed in mixing-length models, or in wavenumber space as in Kolmogorov theory. (The statement that the measurements of the spectrum of laboratory boundary-layer turbulence shown in figure 6.33 agree well with the predictions of the Kolmogorov inertial-range hypothesis and the Heisenberg transfer expression is quite misleading.) Much recent research has shown how far eddy structures in turbulent wakes and pipe flows can persist (for at least 50–100 diameters). These are not academic points because it is important for engineers to know in general where and when turbulence is or is not simply related to the local mean flow.

Chapter 7 on compressible flow is primarily directed at aeronautical applications. The differential and control volume equations for compressible flow, including energy and equations of state, and heat transfer, are derived from first principles. (The details of viscous stresses in compressible flows are given earlier in Chapter 6.) Inviscid, one-dimensional flows with shocks are treated (so the frictional/heat transfer, Fanno/Rayleigh effects are omitted) and then the two-dimensional flows. The discussions of perturbations and the method of characteristics are comprehensive and done with the right approach for engineering students. This chapter is excellent.

There are problems at the end of each chapter and the solutions are at the end of the book. Compressible flow tables are also given at the end, together with formulae of vector calculus. I enjoyed reading the book and I hope that others who also have only a halting knowledge of French will read and consult it too, as well as those in the Francophone world.

J. C. R. HUNT

Non-Newtonian Fluid Mechanics. By G. BÖHME. North-Holland, 1987. 352 pp. Dfl. 180.00 or $80.00.

This relatively compact book is a well-executed translation of the German textbook *Strömungsmechanik nicht-Newtonscher Fluide* published by B. G. Teubner, Stuttgart, 1981. Evidently the book was favourably received by its German language readers and the present camera-ready translation brings it to a wider audience; directed at students in technical universities and practising engineers involved with flow problems of non-Newtonian fluids.

Böhme's goal was evidently to produce a book that could be read without reference to other sources and to develop the theory economically from first principles avoiding '...as much as possible formal arguments, but instead use obvious arguments...'. In fact, he has achieved a tight little book, well-written in an informal style in which every topic is well-motivated and brought to a definite conclusion.

The book is ultimately based on Noll's simple-fluid theory. A simple fluid is a generalization of many special models, but is itself of a special type that depends locally on first spatial velocity gradient, but is global in past time. It is by now well-known that there are two ways to study the rheology of fluids: you can look for a constitutive equation that will apply to a single fluid in many motions or for constitutive expressions which will apply to many fluids for some special motions. The first idea comes up against problems: when the model parameters are adjusted to fit some experiments, they fail to describe others. The second idea is followed by Böhme. He writes down the '...invariant form of the general constitutive equation for viscometric flows...', he looks at the constitutive equations perturbing 'slow and slowly varying flows', the constitutive equations which perturb viscometric flows, and '...the most general law for steady extensional flows'. All these different constitutive equations presumably could apply to one fluid in different motions. In some sense, this second approach could be described as an asymptotic theory of constitutive equations. The stress functional is too general to be of direct use but it is desirable to know what approximate theories are supposed to approximate.

The asymptotic approach to constitutive equations has its own problems, because we are in the dark when it comes to complex flows. Böhme's book is not the one to learn about different constitutive models for complex flows.

To achieve economy of presentation in a unified and strictly business-like manner, Böhme made a courageous (or foolish) decision. In a subject noted for devotion to history and attribution, he decided not to give anybody credit; not me, you, or even himself. Not claiming priority for your own original work is a form of saintliness rare even in religion, not to mention fluid mechanics. Obviously this approach has the defect that students receive no instruction about how and where to go more deeply. Perhaps some excitement of attribution is contained in the sense one receives of an intellectually active subject in which discovery and even controversy are not dead. This loss may be too high a price to pay for economy of presentation.

The mathematical level of the book is not overly demanding, perhaps less demanding than other recently published works on rheological fluid mechanics.

Though Böhme's book does rest on first principles, he has achieved major simplifications. There are on the average three or four fairly short equations on a page, attractively set off from the explanatory text. The book unfolds in a logical, easy manner. The connections between experiments and engineering applications are well drawn. The assumptions used in approximate analysis seem to be clearly stated.

There are nine chapters. The first develops some elementary ideas from continuum mechanics, with particular attention to the kinematic relations which arise on special viscometric flows. The second chapter is about the modelling of the shear stress function and the two normal stress functions using the somewhat controversial idea of similarity for these functions of the rate of shear with respect to changes of molecular weight, concentration, pressure, or temperature. The third chapter treats flows that appear to be governed by the shear stress function; this includes some processing flows which can be described by combined Couette and Poiseuille flow, screw extruders, rollers and journal bearings.

In the fourth chapter the author considers some effects of normal stresses with simplified analyses of cone and plate flow, rod climbing, die swell and secondary motions in flow down straight pipes. In this chapter and again in chapter 9 he takes notice of a critical radius which decides the competition between normal stresses and inertia. In rod climbing, the fluid will sink outside and climb inside a radius given by the ratio of a combination of a second-order constants (usually called the climbing constant) to the density. A critical radius appears in the secondary motions driven by rotating spheres, rods, cones, and other axisymmetric objects with two sets of counter-rotating eddies, inertia dominating outside and normal stresses inside. I think that this phenomenon and its cause are one of the main signatures of non-Newtonian flow at low speeds, and it is well explained at the fundamental mathematical level by second-order fluid theory. My explanation is as follows: let L be a typical length for gradients in the flow and U a typical velocity which in part of my argument I shall put equal to ωL. Then the gradient of inertia scales like $\rho U^2/L$ and the divergence of the normal stresses like $\alpha U^2/L^3$, and the ratio $\alpha/\rho L^2$ is order one when the two effects are in competition. I go further and note that when $L \to 0$, $\rho U^2 \sim \rho\omega^2 L^2$ tends to zero but $\alpha U^2/L^2 \sim \alpha\omega^2$ remains finite. Rods of vanishing radius still induce a finite climb, normal stress in holes produce pressure readings no matter how small the holes, low-speed jets swell no matter how small the diameter, etc.

In chapter five, Böhme treats unsteady flows, step strains, creep, plane harmonic waves and Stokes's first problem. The interesting analysis of the tuning of a shock absorber was new to me as was the analysis of nonlinear effects in unsteady pipe flow based on the expansion of terms through fourth order for slow, nearly steady motions. In Chapter 6 he treats the problem of perturbed viscometric flow by using the theory of Pipkin and Owen giving results, not published in other books, which were developed in the group of the late Professor Becker at Darmstadt. Boundary layers, lubricating films, and the stability of plane shear flows are discussed. Extensional flows, uniaxial, biaxial, and plane extensions, are treated in Chapter 7 with applications to fibre drawing and the oscillating bubble. Chapter 8 is about special rheological laws, single integral and Oldroyd differential models being treated briefly. This chapter also contains an interesting discussion of slow steady flow, in some sense out of place here because it is an asymptotic rather than a special theory.

All of the many problems discussed in Chapter 9 on 'secondary flows' can be done in the context of slow motion expansions, but are nicely understood using the method of primary and secondary motion. The primary flow is Newtonian and usually simple, one component of velocity depending on one spatial variable. Secondary motions arise from this in the usual way, as a perturbation. The author treats different problems, rotationally axisymmetric flow, where the perturbing secondary flow arises at second order, and other problems where the secondary motions arise at higher order.

There are other more comprehensive textbooks which cover the same material as this volume. Perhaps the approach taken in other volumes is more eclectic. The unique feature of Böhme's book is that it successfully attains a circumscribed goal: a modelling for engineering applications. The restricted range of problems considered in the book, and the lack of references, make it unsuitable as a basic reference for research and applications, but different books have different aims.

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